STRESS

1 Stress 3

1.3 Stress Analysis

- The sectioned area to be subdivided into small areas, such as ΔA shown in Fig.1–9*a*.
- A typical finite yet very small force ΔF , acting on ΔA.
- We will replace ΔF by its *three components* , namely, ΔF_x , ΔF_y, **and** ΔF_z, which are taken tangent, tangent, and normal to the area, respectively.

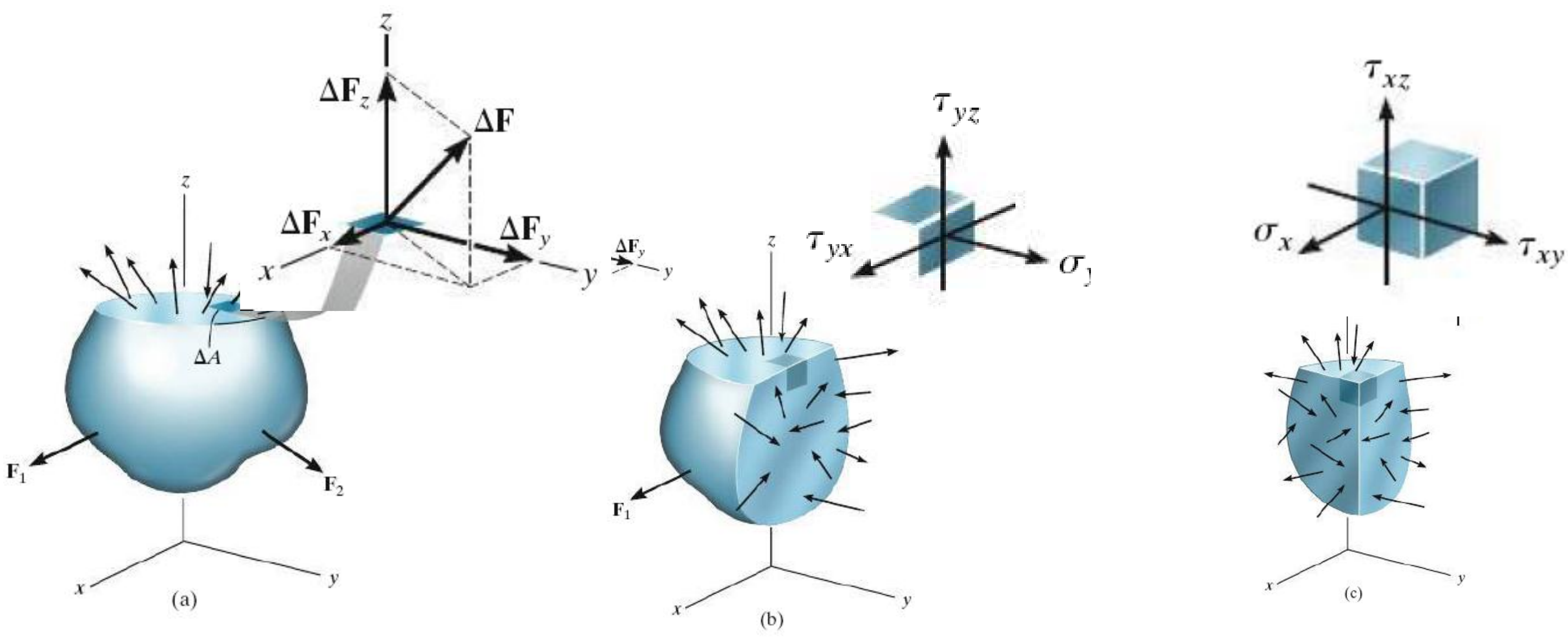


Fig. 1–9

STRESS

- As ΔA approaches zero, so do ΔF and its components; however, the quotient of the force and area will, in general, approach a finite limit.
- This quotient is called ***stress*** **.**
- ***Stress*** describes the *intensity of the internal force* acting on a *specific plane* (area) passing through a point.

❖ ***Normal Stress σ (sigma).*** The *intensity* of the force acting normal to ΔA. Since ΔF_z is normal to the area then:-

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

If the normal force or stress "pulls" on ΔA , it is referred to as ***tensile stress*** , whereas if it "pushes" on ΔA it is called ***compressive stress*** .

❖ *Shear Stress τ(Tau).* The intensity of force acting tangent to ΔA. Here we have shear stress components,

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$

General State of Stress.

This state of stress is then characterized by three components acting on each face of the element, Fig. 1–11 .

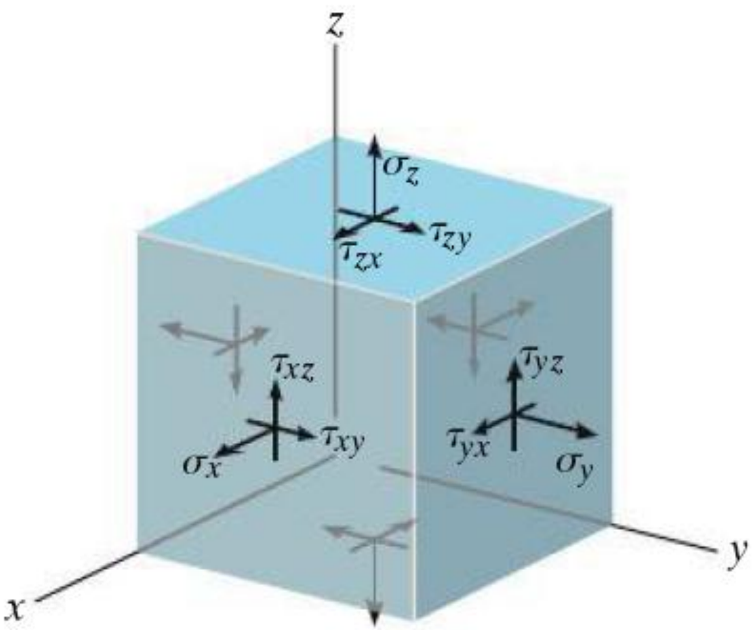


Fig. 1–11

1.4 Average Normal Stress in an Axially Loaded Bar

- As a result, each small area ΔA on the cross section is subjected to a force $\Delta F = \sigma \, \Delta A$
- the *sum* of these forces acting over the entire cross-sectional area must be equivalent to the internal resultant force **P** at the section.
- If we let $\Delta A = \int dA$ and therefore $\Delta F = \int dF$, then, recognizing σ is constant, we have

$$\int dF = \int_A \sigma \, dA$$

$$P = \sigma A$$

$$\boxed{\sigma = \frac{P}{A}}$$

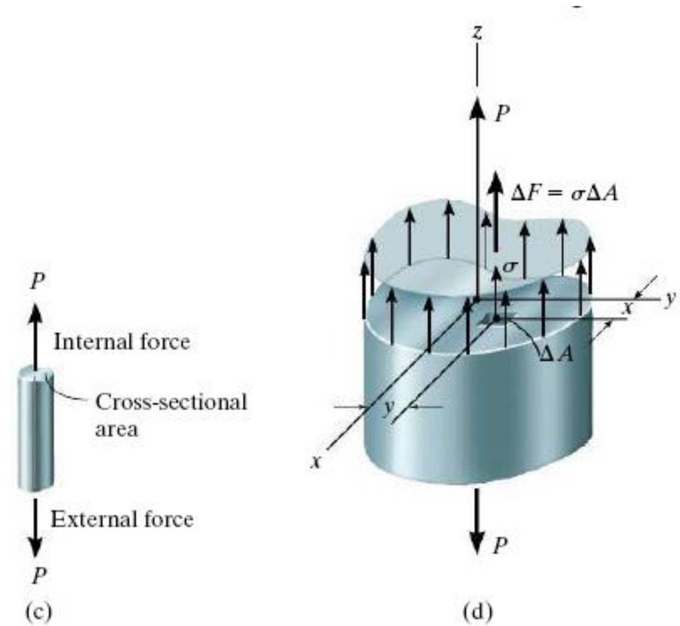


Fig. 1–12

STRESS

$$\sigma = \frac{P}{A}$$

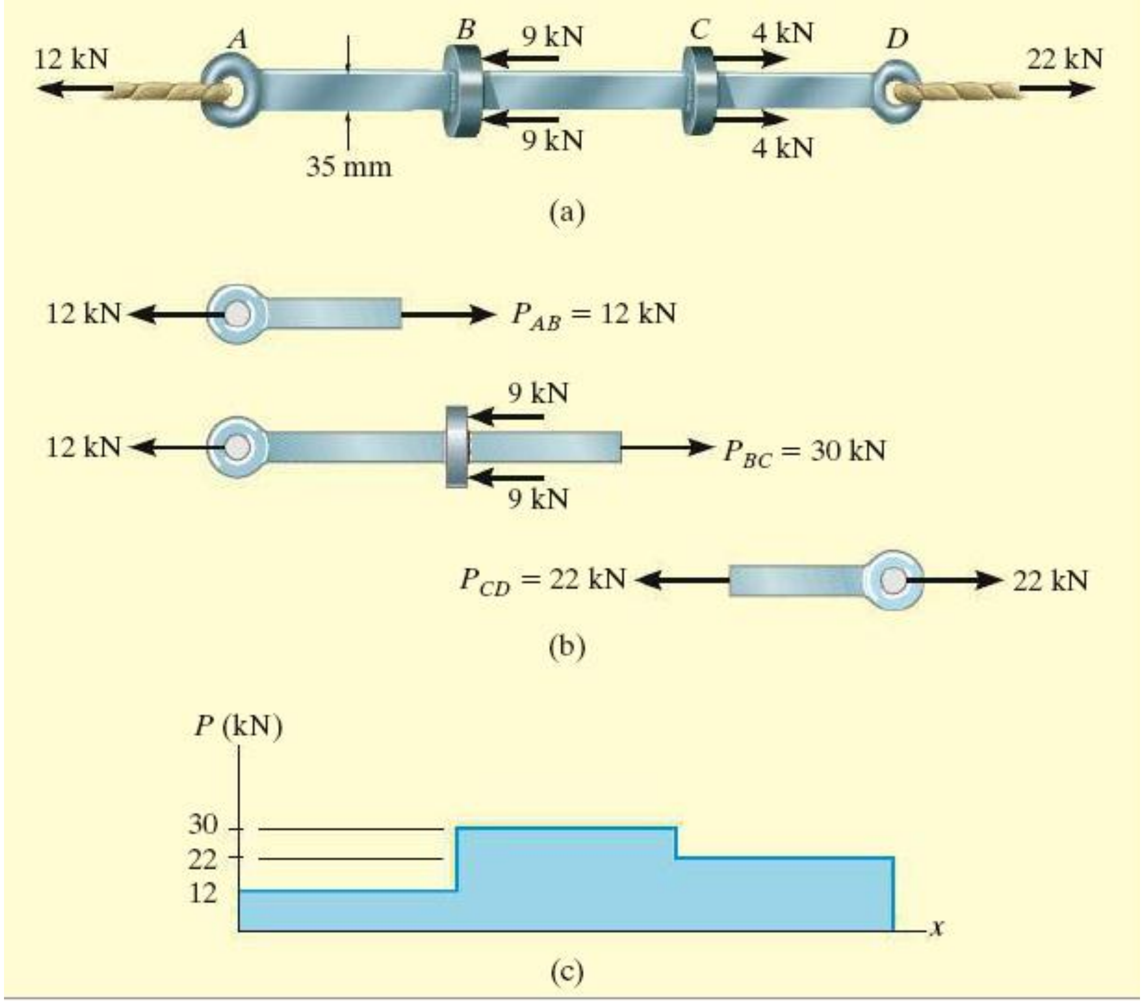
Here:-

$\boldsymbol{\sigma}$ = average normal stress at any point on the cross-sectional area

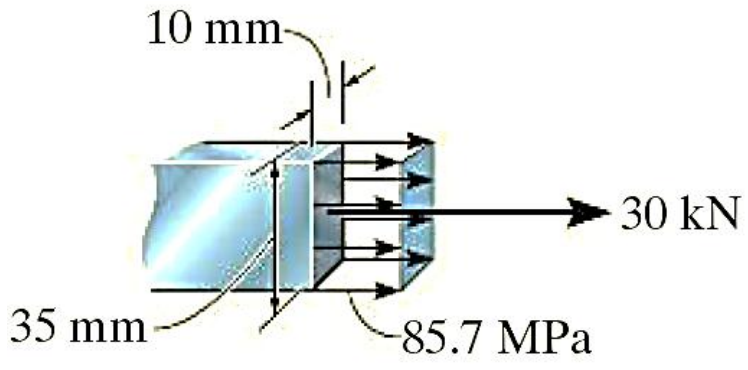
$\boldsymbol{P}$ = *internal resultant normal force*, which acts through the *centroid* of the cross-sectional area. P is determined using the method of sections and the equations of equilibrium

$\boldsymbol{A}$ = cross-sectional area of the bar where $\boldsymbol{\sigma}$ is determined

EX1:- The bar in Fig. 1–15 *a* has a constant width of **35 mm** and a thickness of **10 mm.** Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



Average Normal Stress.

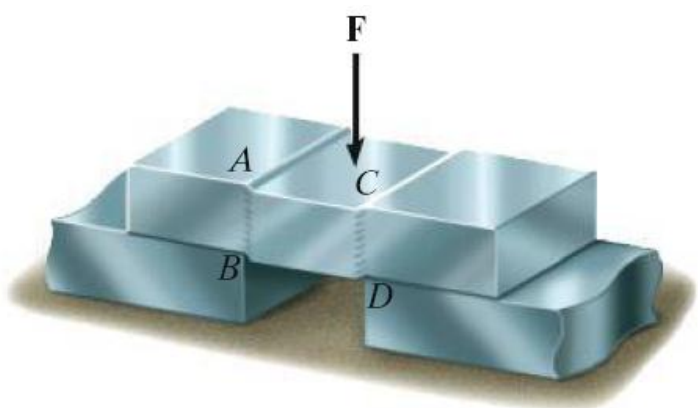


$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa}$$

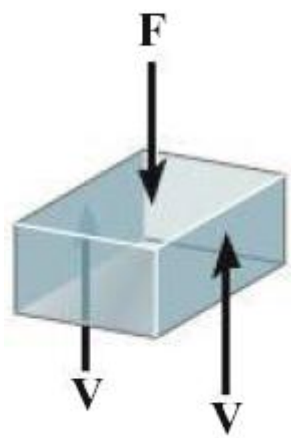
Graphically, the *volume* represented by this distribution of stress is equivalent to the load of 30 kN; that is, 30 kN = (85.7 MPa)(35 mm)(10 mm).

1.5 Average Shear Stress.

- If the supports are considered rigid, and **F** is large enough, it will cause the material of the bar to deform and fail along the planes identified by *AB* and *CD* .
- A free-body diagram of the unsupported center segment of the bar, Fig. 1–19 *b* , indicates that the shear force ***V* = *F*\2** must be applied at each section to hold the segment in equilibrium.



(a)



(b)

- The ***average shear stress*** distributed over each sectioned area that develops this shear force is defined by

$$\tau_{avg} = \frac{V}{A}$$

- ✓ τ_{Ave} = average shear stress at the section, which is assumed to be the *same* at each point located on the section
- ✓ V = internal resultant shear force on the section determined from the equations of equilibrium
- ✓ A = area at the section

- The distribution of average shear stress acting over the sections is shown in Fig. 1–19 *c*. Notice that τ_{Ave} is in the *same direction* as **V** , since
- the shear stress must create associated forces all of which contribute to
- the internal resultant force **V** at the section.

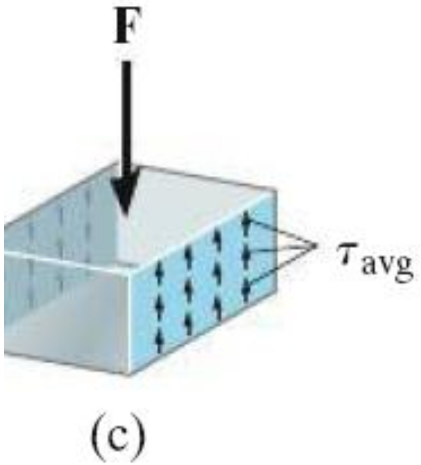
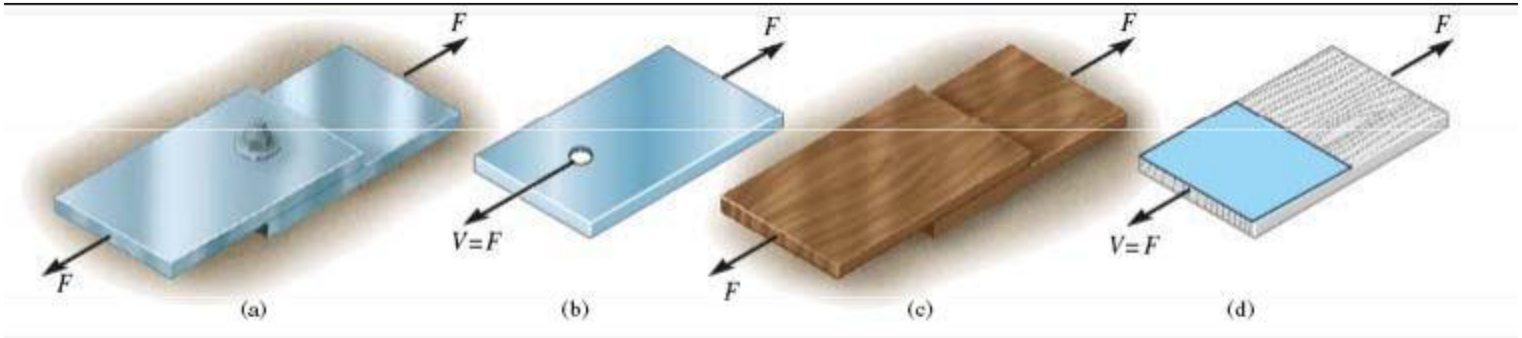


Fig. 1–19

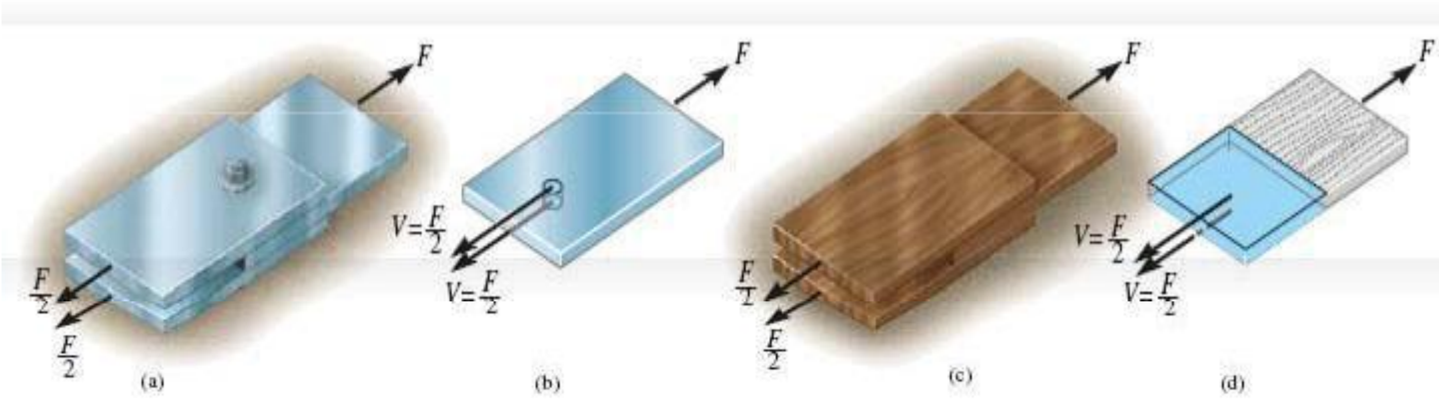
STRESS



Shear stress on bolt
τ =F/A = F/ πr2
Where; r is the radius of the bolt

Shear stress on **bonded area.**
τ =F/bc.
Where; (bc) is the area of contact subjected to the shear force

STRESS



$\tau=(F/2)/A=F/2A$

Where; **A** is the parallel area of the **bolt** subjected to shear force

τ=(F/٢)/A=F/٢A

$A = cb$

Where; **A** is the parallel area of **the bonded region** subjected to shear force

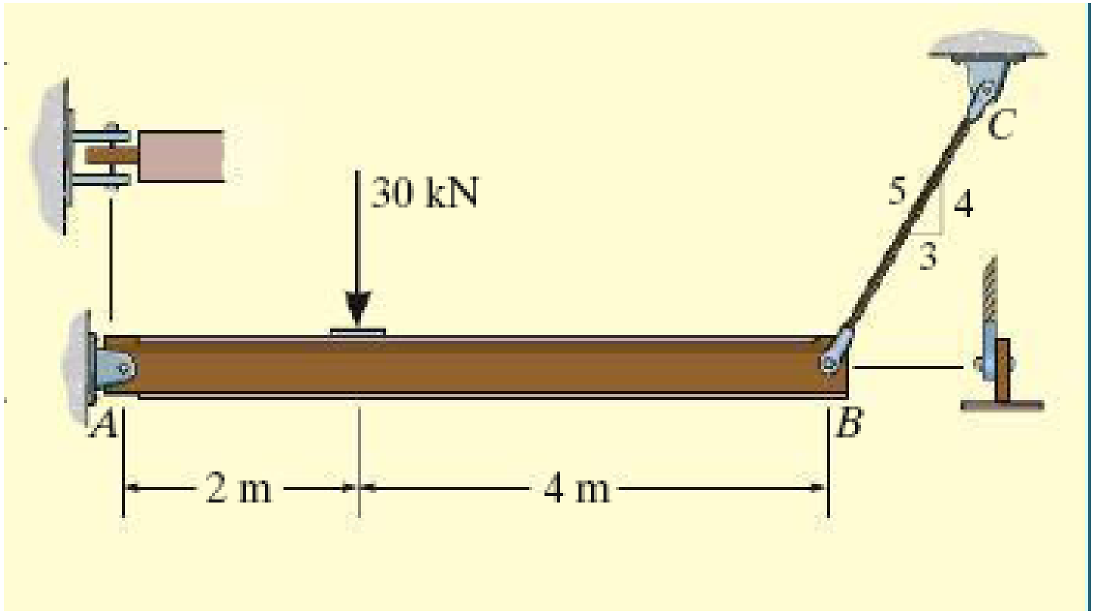
STRESS

The pin *A* used to connect the linkage of this tractor is subjected to *double shear* because shearing stresses occur on the surface of the pin at *B* and *C*. See Fig 1–21*c*.

Ex1:-Determine the average shear stress in the 20-mm-diameter pin at *A* and the 30-mm-diameter pin at *B* that support the beam in Fig. 1–21 *a* .

Solution

Internal Loadings. The forces on the pins can be obtained by considering the equilibrium of the beam, Fig. 1–21 *b* .



STRESS

$$\circlearrowleft + \Sigma M_A = 0;$$

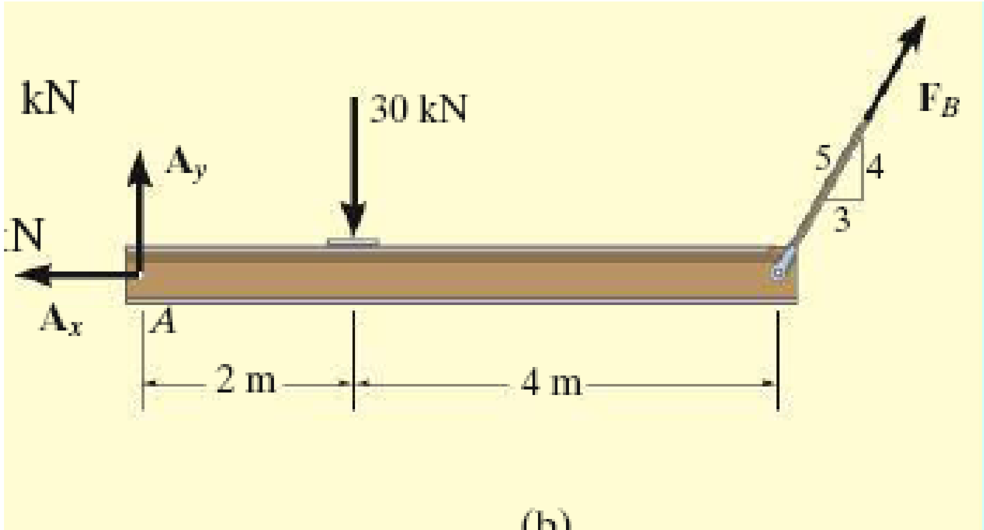
$$F_B\left(\frac{4}{5}\right)(6\text{ m}) - 30\text{ kN}(2\text{ m}) = 0 \qquad F_B = 12.5\text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad (12.5\text{ kN})\left(\frac{3}{5}\right) - A_x = 0 \qquad A_x = 7.50\text{ kN}$$

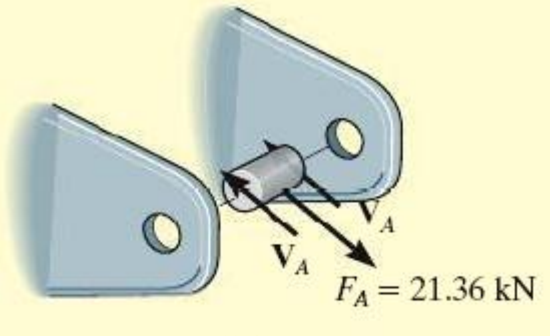
$$+\uparrow \Sigma F_y = 0; \quad A_y + (12.5\text{ kN})\left(\frac{4}{5}\right) - 30\text{ kN} = 0 \quad A_y = 20\text{ kN}$$

Thus, the resultant force acting on pin A is

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{(7.50\text{ kN})^2 + (20\text{ kN})^2} = 21.36\text{ kN}$$



$$V_A = \frac{F_A}{2} = \frac{21.36 \text{ kN}}{2} = 10.68 \text{ kN}$$



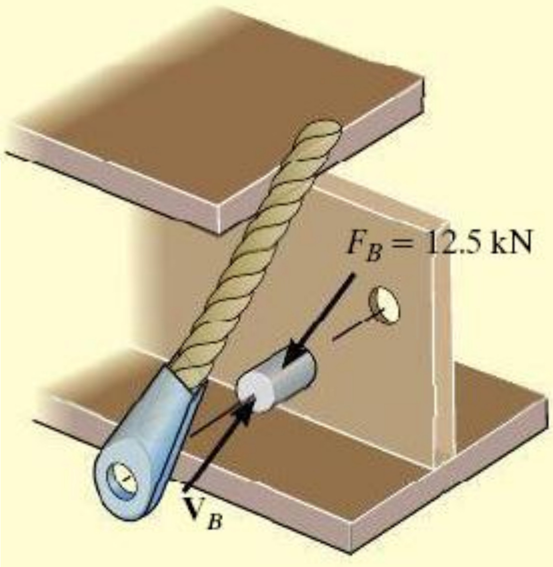
(c)

$$V_B = F_B = 12.5 \text{ kN}$$

Average Shear Stress.

$$(\tau_A)_{avg} = \frac{V_A}{A_A} = \frac{10.68(10^3) \text{ N}}{\frac{\pi}{4}(0.02 \text{ m})^2} = 34.0 \text{ MPa}$$

$$(\tau_B)_{avg} = \frac{V_B}{A_B} = \frac{12.5(10^3) \text{ N}}{\frac{\pi}{4}(0.03 \text{ m})^2} = 17.7 \text{ MPa}$$



(d)

1.6 Allowable stress design.

- To ensure the safety of a structural member or mechanical element, it is necessary to restrict the applied load to one that is less than the load the member (or element) can fully support.

Cranes are often supported using bearing pads to give them stability. Care must be taken not to crush the supporting surface, due to the large bearing stress developed between the pad and the surface.

- One method of specifying the allowable load for a member is to use a number called the factor of safety.

- The ***factor of safety*** (F.S.) is a ratio of the failure load F_{Fail} to the allowable load F_{allow} .

- Here F_{Fail} is found from experimental testing of the material, and the factor of safety is selected based on experience

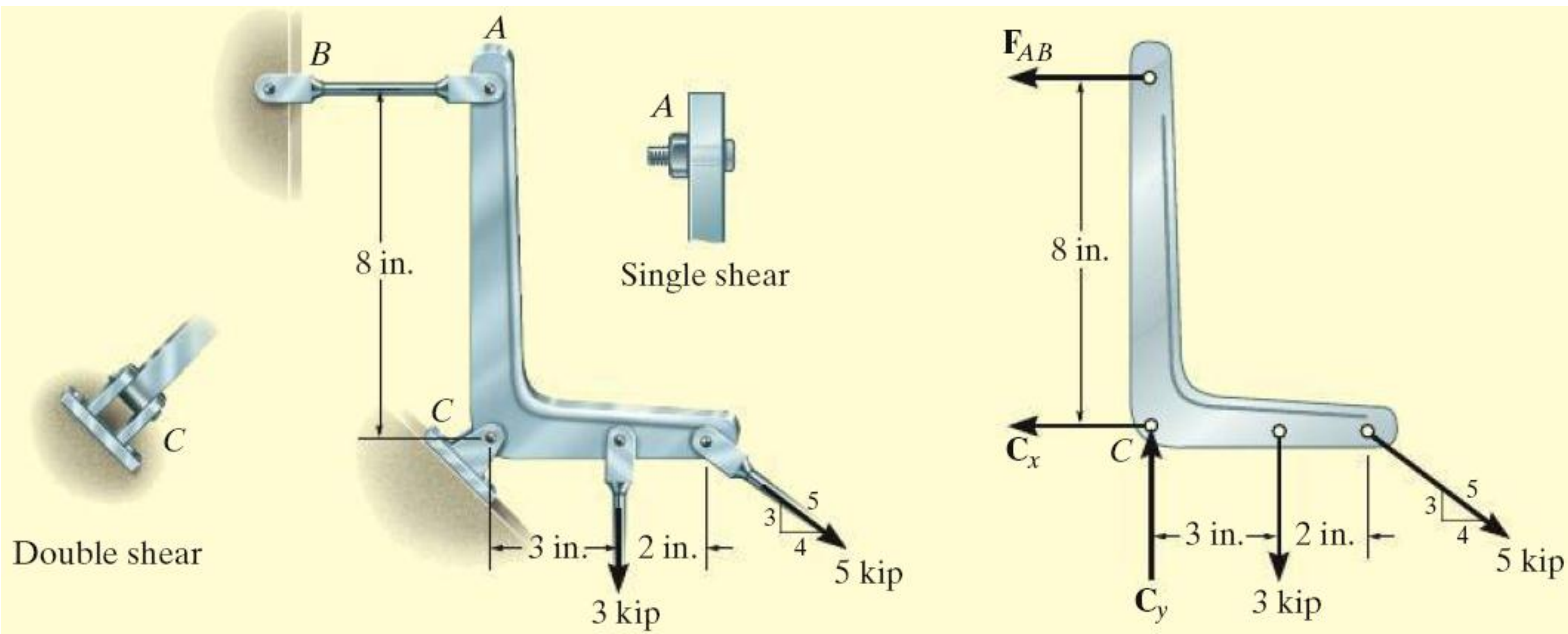
$$\text{F.S.} = \frac{F_{\text{fail}}}{F_{\text{allow}}}$$

- If the load applied to the member is ***linearly related*** to the stress developed within the member, as in the case of using $\boldsymbol{\sigma = P/A}$ and $\boldsymbol{\tau_{avg} = V/A}$, then
- we can also express the factor of safety as a ratio of the failure stress $\boldsymbol{\sigma_{Fail}}$ **(or** $\boldsymbol{\tau_{Fail}}$ **)** to the *allowable stress* $\boldsymbol{\sigma_{allow}}$ **(or** $\boldsymbol{\tau_{allow}}$ **).**
- Here the area A will cancel and so,

$$\text{F.S.} = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}} \qquad (1\text{–}9)$$

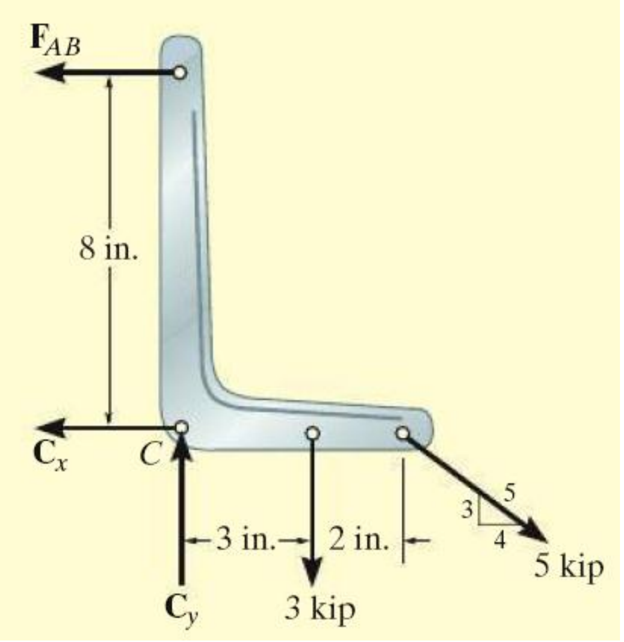
$$\text{F.S.} = \frac{\tau_{\text{fail}}}{\tau_{\text{allow}}} \qquad (1\text{–}10)$$

Ex1 :- The control arm is subjected to the loading shown in Fig. 1–25 a . Determine to the nearest $\frac{1}{4}$ **in**. the required diameters of the steel pins at **A** and **C** if the allowable shear stress for the steel is τ_{allow}= **8 ksi**.



Solution:-

Pin forces. A free-body diagram of the arm is shown in fig. 1-25b. For equilibrium we have



$$\circlearrowleft + \Sigma M_C = 0; \quad F_{AB}(8 \text{ in.}) - 3 \text{ kip } (3 \text{ in.}) - 5 \text{ kip } \left(\tfrac{3}{5}\right)(5 \text{ in.}) = 0$$

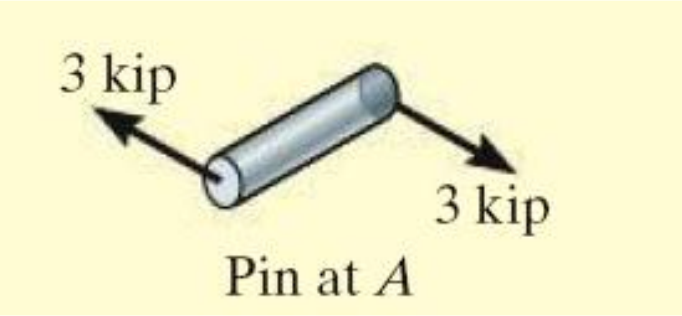
$$F_{AB} = 3 \text{ kip}$$

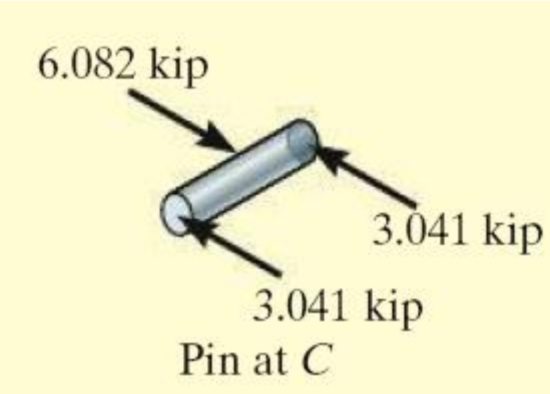
$$\xrightarrow{+} \Sigma F_x = 0; \quad -3 \text{ kip} - C_x + 5 \text{ kip } \left(\tfrac{4}{5}\right) = 0 \qquad C_x = 1 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 3 \text{ kip} - 5 \text{ kip}\left(\tfrac{3}{5}\right) = 0 \qquad C_y = 6 \text{ kip}$$

The pin at C resists the resultant force at C, which is

$$F_C = \sqrt{(1 \text{ kip})^2 + (6 \text{ kip})^2} = 6.083 \text{ kip}$$





Pin A. This pin is subjected to single shear, Fig. 1–25*c*, so that

$$A = \frac{V}{\tau_{\text{allow}}}; \qquad \pi\left(\frac{d_A}{2}\right)^2 = \frac{3\text{ kip}}{8\text{ kip/in}^2}; \qquad d_A = 0.691\text{ in.}$$

Use $d_A = \frac{3}{4}$ in. *Ans.*

Pin C. Since this pin is subjected to *double shear*, a shear force of 3.041 kip acts over its cross-sectional area *between* the arm and each supporting leaf for the pin, Fig. 1–25*d*. We have

$$A = \frac{V}{\tau_{\text{allow}}}; \qquad \pi\left(\frac{d_C}{2}\right)^2 = \frac{3.041\text{ kip}}{8\text{ kip/in}^2}; \qquad d_C = 0.696\text{ in.}$$

Use $d_C = \frac{3}{4}$ in. *Ans.*